

Class 12 Maths - Application of Derivatives

JEE track | Short Notes + 5 CBSE-based questions + 5 JEE Main PYQ-based questions with solutions

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Format: Quick revision + solved practice	Chapter scope: Class 12 Maths

1. Quick Short Notes

- Derivative represents instantaneous rate of change and slope of tangent to the curve.
- If $f'(x) > 0$ on an interval, f is increasing there; if $f'(x) < 0$, f is decreasing.
- Critical points occur where $f'(x) = 0$ or f' is not defined.
- A sign change from $+$ to $-$ in $f'(x)$ gives local maximum; from $-$ to $+$ gives local minimum.
- Equation of tangent at $x = a$: $y - f(a) = f'(a)(x - a)$.
- Equation of normal at $x = a$ uses slope $-1/f'(a)$, provided $f'(a) \neq 0$.
- Second derivative test: if $f'(a) = 0$ and $f''(a) > 0$, local minimum; if $f''(a) < 0$, local maximum.
- Applications include optimisation of area, cost, volume and distance problems.
- Approximation can be done using $dy = f'(x) dx$ for small changes.
- Exam tip: after finding critical points, always verify nature using sign test or second derivative test.

2. CBSE-based Board Practice

Q1. Find the intervals where $f(x) = x^2 - 4x + 3$ is increasing and decreasing.

Solution: $f'(x) = 2x - 4 = 2(x - 2)$. So $f'(x) < 0$ for $x < 2$ and $f'(x) > 0$ for $x > 2$. Hence decreasing on $(-\infty, 2)$ and increasing on $(2, \infty)$.

Q2. Find the local maxima and minima of $f(x) = x^3 - 3x$.

Solution: $f'(x) = 3x^2 - 3 = 3(x - 1)(x + 1)$. Critical points $x = -1, 1$. $f''(x) = 6x$. At $x = -1$, $f'' < 0$ so local maximum = $f(-1) = 2$. At $x = 1$, $f'' > 0$ so local minimum = $f(1) = -2$.

Q3. Find the equation of tangent to $y = x^2$ at $x = 2$.

Solution: $dy/dx = 2x$, so slope at $x = 2$ is 4. Point is $(2, 4)$. Tangent: $y - 4 = 4(x - 2)$, i.e. $y = 4x - 4$.

Q4. Find the equation of normal to $y = x^2$ at $x = 1$.

Solution: Tangent slope at $x = 1$ is 2, so normal slope = $-1/2$. Point is $(1, 1)$. Normal: $y - 1 = (-1/2)(x - 1)$.

Q5. Find the minimum value of $x + 9/x$ for $x > 0$.

Solution: Let $f(x) = x + 9/x$. Then $f'(x) = 1 - 9/x^2 = 0$ gives $x = 3$. Since $f''(x) = 18/x^3 > 0$, minimum occurs at $x = 3$. Minimum value = $3 + 3 = 6$.

3. JEE Main PYQ-based Practice

Q1. How many tangents to the curve $y = x^2$ are parallel to the line $y = 4x + 1$? Also write the tangent.

Solution: Parallel lines have same slope 4. For $y = x^2$, derivative = $2x$. Set $2x = 4$ to get $x = 2$. Hence only one tangent exists, and it is $y = 4x - 4$.

Q2. Find the minimum value of $x + 1/x$ for $x > 0$.

Solution: $f'(x) = 1 - 1/x^2 = 0$ gives $x = 1$. Since $f''(x) = 2/x^3 > 0$, minimum value = $1 + 1 = 2$.

Q3. If $f'(x) > 0$ on an interval, what can be concluded about f on that interval?

Solution: The function is strictly increasing on that interval.

Q4. Find the tangent to $y = x^3$ at $x = 1$ and its intercept on y-axis.

Solution: $dy/dx = 3x^2$, so slope at $x = 1$ is 3. Point is (1,1). Tangent: $y - 1 = 3(x - 1)$, so $y = 3x - 2$. Y-intercept is -2.

Q5. Find the maximum value of $\sin x + \cos x$.

Solution: $\sin x + \cos x = \sqrt{2} \sin(x + 45 \text{ degrees})$. Hence maximum value is $\sqrt{2}$.

Practice tip: First revise the short notes, then attempt CBSE board questions in written format, and finally solve the exam-specific section in timed mode.